Stochastic, Integral and Combinatorial Geometry

International Conference in honor of Professor Rouben Ambartzumian's 80th birthday

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Organized by:
-- Russian Armenian University, Yerevan
-- Institute of Mathematics of National Academy of Science of RA

ABSTRACTS

1. Armen Sergeev (Steklov Mathematical Institute, Moscow)

Spinor Geometry and Seiberg-Witten Equations

Seiberg–Witten equations invented at the end of XX-th century play a key role in the study of compact Riemannian 4-manifolds. They are defined in terms of spinor geometry, more precisely, of Spin^c-structure which exists on any such manifold. In the case of Kahler surfaces, i.e. Riemannian 4-manifolds provided with a compatible complex structure, the moduli space of their solutions can be identified with the space of holomorphic curves on the surface lying in a given topological class. In a more interesting case of simplistic 4-manifolds it is possible to introduce the adiabatic limit procedure. In this limit solutions of the original Seiberg–Witten equations concentrate in a neighborhood of some pseudoholomorphic curve belonging to a given topological class. Simultaneously, the original Seiberg–Witten equations reduce to the adiabatic equation on the limiting pseudoholomorphic curve which may be considered as a nonlinear $\bar{\partial}$-equation.

2. Werner Nagel (Jena University, Germany)

Markov processes of tessellations that are generated by cell division:

In the 80th of the last century, Rouben Ambartzumian posed a problem: Let X be a random tessellation, e.g. a planar Poisson line tessellation. Then, into each cell P1, P2, . . . of X an independent copy X1, X2, . . . of the random tessellation, is nested. Thus the original cells are subdivided, and this yields a new tessellation. Then continue with the procedure of nesting independent copies into the extant cells. After each step, rescale the tessellation appropriately. Question: Is there some limiting tessellation for this iterated nesting of independent tessellations? Together with Joseph Mecke and Viola Weiß we worked on this problem, and we found that, under some regularity conditions, there is a unique limit – we called it the STIT tessellation (stochastically Stable under the operation of Iteration/nesting of tessellations). The model of STIT tessellations will be presented in detail as well as some of the numerous results which are shown for it so far. In the second part of the talk a generalization and modifications of the STIT model are described. Those are motivated by applications to crack structures as they appear in nanotechnology, materials science, soft matter, and geology.

References:

W. Nagel and V. Weiß, Crack STIT tessellations: characterization of stationary random tessellations stable with respect to iteration. Adv. Appl. Prob. 37 (2005), pp. 859-883.

R. León, W. Nagel, J. Ohser and S. Arscott, Modeling crack patterns by modified STIT tessellations. Image Anal. Stereol. 39 (2020), pp. 33-46. (open access)

3. Rafik Aramyan (Russian-Armenian University, Armenia)

Local reconstruction from the spherical mean Radon transform

We apply a new approach for the inversion of the spherical mean Radon transform in 3D with detectors on a plane. We present a new iterative formula that gives an algorithm to recover an unknown function supported completely on one side of a plane E from its spherical means over spheres centered on the plane E. Our reconstruction formula has the benefit of being local. Such an inversion is required in problems of thermo-and photo-acoustic tomography, ultrasound reflection tomography, radar imaging, and a few other. In [1] was found an inversion of the spherical mean Radon transform in 2D with detectors on a line, which has the benefit of being local.

1. R. Aramyan "To local reconstruction from the spherical mean Radon transform" Journal of Mathematical Analysis and Applications, vol. vol. 470 (1), pp. 102-117, 2019.

4. Dietrich Stoyan (TU Bergakademie Freiberg, Germany)

Stochastic Geometry in Arts and Architecture

Modern computer graphics likes to use models of stochastic geometry for generating interesting patterns. These patterns, preferably tessellations and germ-grain models, are so becoming popular also for laymen and are today used in various ways, even for textiles in everyday use. The talk gives a series of examples. Perhaps most interesting are examples from architecture, in situations where asymmetric areas have to be decorated. A statistician is attempted to analyse such patterns with methods of spatial statistics.

5. Linda Khachatryan, Boris Nahapetian (Institute of Math. NAS of Armenia)

Combinatorial approach to the description of random fields

The study of random field (probability measure on an infinite-dimensional space of its realizations), as a rule, is performed through some simpler object: a system of conditional or unconditional finite-dimensional distributions, correlation functions, etc.

The theory of correlation functions is widely used in statistical physics since it allows the description of states of an infinite physical system. Ruben Ambartzumian made a significant contribution to this theory. In his pioneering work with Sukiasian [1], he developed the combinatorial approach to the characterization of limiting correlation functions for point processes. This approach received recognition and was further used in the works of well-known mathematicians (see, for example, [4] and [5]).

In [3] applying the combinatorial approach, the problem of the description of random fields was solved in the discrete case of lattice systems with state space {0,1}. This was done in terms of the newly introduced notion of P-function, which can be interpreted as a system of limiting correlation functions.

The present talk (see also [4]) concentrates on the further development of the combinatorial approach to the description of lattice random fields with general (Polish) state space. We suggest a method of construction of P-functions, which can be applied in the theory of Gibbs random fields.

References

[1] R.V. Ambartzumian, H.S. Sukiasian, Inclusion-exclusion and point processes, Acta Appl. Math. 22, 1991, 15-31

[2] S. Dachian, B.S. Nahapetian, Inclusion-Exclusion description of random fields, J. Contemp. Math. Anal. Arm. Acad. Sci. 30 (6), 1995, 50-61

[3] L.A. Khachatryan, B.S. Nahapetian, Combinatorial approach to the description of random fields, Lobachevskii J. Math. 42 (10), 2021, 2337–2347

[4] L. Koralov, An inverse problem for Gibbs fields, CRM Proceedings and Lecture Notes 42, 2007, 299-307

[5] T. Kuna, J.L. Lebowitz, E.R. Speer, Realizability of Point Processes, J. Stat. Phys. 129 (3), 2007, 417-439

6. Hayk S. Sukiasyan (Institute of Math. NAS of Armenia)

Random Sections of Polyhedron

In Euclidean space R^3 we consider a family of convex octahedra. Octahedron is a polyhedron with 8 triangular faces, 12 edges and 6 vertices. Each vertex is the common point of four edges. The sections of convex octahedra by random planes are studied. The distribution of random planes is generated by the measure of the
planes, which is invariant with respect to Euclidean motions. The probabilities P_n of events that these sections are n-gon are considered, n=4,6.

Using R.Ambartzumian's combinatorial formula the probabilities P_n are calculated.
It is proved that P_4 reaches his minimal value and P_6 reaches his maximum value for regular octahedra. The asymptotic behavior of these probabilities
is investigated as the altitude trends to zero or infinity.
For these probabilities we obtain the inequalities, which hold for any altitude.

6. Elen Aramyan (Russian-Armenian University, Armenia)

Reconstruction of convex polygon by its chord length distributions

It is known that every planar bounded convex domain is determined uniquely within all planar convex domains by its orientation dependent chord length distribution functions, up to translation and reflections (see [1]). The problem of reconstruction of a convex domain by its orientation dependent chord length distribution functions was still open.

Here we present an algorithm for reconstruction of a convex planar polygon from its orientation dependent chord length distribution functions.

1. G. Bianchi and G. Averkov, ``Confirmation on Matheron's conjecture on the covariogram of a planar convex body'', Journal of the European Mathematical Society, vol. 11, pp. 1187-1202, 2009.